

$L_\mu - L_\tau$ gauge-boson production from LFV τ decays at Belle IIChuan-Hung Chen^{1,*} and Takaaki Nomura^{2,†}¹*Department of Physics, National Cheng-Kung University, Tainan 70101, Taiwan*²*School of Physics, KIAS, Seoul 02455, Korea*

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Abstract

$L_\mu - L_\tau$ gauge boson (Z') with a mass in the MeV to GeV region can resolve not only the muon $g-2$ excess, but also the gap in the high-energy cosmic neutrino spectrum at IceCube. It was recently proposed that such a light gauge boson can be detected at the Belle II experiment with a luminosity of 50 ab^{-1} by the $e^+e^- \rightarrow \gamma + \cancel{E}$ process through the kinetic mixing with the photon, where the missing energy \cancel{E} is from the $Z' \rightarrow \bar{\nu}\nu$ decays. We study the phenomenological implications when a pair of singlet vector-like leptons carrying different $L_\mu - L_\tau$ charges are included, and a complex singlet scalar (ϕ_S) is introduced to accomplish the spontaneous $U(1)_{L_\mu - L_\tau}$ symmetry breaking. It is found that the extension leads to several phenomena of interest, and they include: (i) branching ratio (BR) for $h \rightarrow \mu\tau$ can be of the order of 10^{-3} ; (ii) ϕ_S -mediated muon $g-2$ can be of the order of 10×10^{-10} ; (iii) BR for $\tau \rightarrow \mu\phi_S^* \rightarrow \mu Z' Z'$ can be 10^{-8} ; and (iv) kinetic mixing between the Z' boson and photon is sensitive to the relative heavy lepton masses. The predicted BRs for $\tau \rightarrow (3\mu + \cancel{E}, 5\mu)$ through the leptonic Z' decays can reach a level of 10^{-9} , in which the results fall within the sensitivity of Belle II in the search for the rare tau decays.

*Electronic address: physchen@mail.ncku.edu.tw

†Electronic address: nomura@kias.re.kr

A Z' gauge boson, dictated by an anomaly-free $U(1)_{L_\mu-L_\tau}$ gauge symmetry [1, 2], has been broadly studied. Especially, the Z' boson with a mass in the range between MeV and GeV can help explain the observed anomalies, such as muon anomalous magnetic moment (muon $g - 2$), and deficiency in the high-energy cosmic neutrino spectrum reported by IceCube [3–7]. In addition, the $U(1)_{\mu-\tau} \equiv U(1)_{L_\mu-L_\tau}$ gauge model can also resolve the largely unexpected lepton-flavor nonuniversality in the semileptonic B decays [8–10] and Higgs h lepton flavor violating (FLV) decays [9, 11–13]; dark matter physics and/or neutrino mass is also discussed [14–18].

Recently, there are some progress on detecting the light Z' boson in experiments and on limiting the ranges of the Z' mass $m_{Z'}$ and gauge coupling $g_{Z'}$, which are used to fit the muon $g - 2$ anomaly. For instance, according to the neutrino trident production processes, which were measured by CHARM-II collaboration [19] and CCFR collaboration [20], it was shown that $m_{Z'} \gtrsim 400$ MeV and $g_{Z'} > \text{few} \times 10^{-3}$ are excluded [21]. Based on the observation of the cross-section for the $e^+e^- \rightarrow \mu^+\mu^-Z'$, $Z' \rightarrow \mu^+\mu^-$ channel, which was recently measured by BABAR collaboration at 90% confidence level (CL) [22], the bound of $g_{Z'} < 0.7 \times 10^{-3}$ at $m_{Z'} \approx 0.22$ GeV is obtained. The ranges of $m_{Z'} \subset (1, 10)$ MeV and $g_{Z'} \subset (0.1, 1) \times 10^{-3}$ are badly narrowed [23] by the measurement of ${}^7\text{Be}$ solar neutrino scattering off the electron in Borexino experiment [24], where the ν - e scattering occurs through the loop-induced kinetic mixing between the electromagnetic and Z' gauge fields. Although the $m_{Z'}$ and $g_{Z'}$ parameter spaces for explaining the muon $g - 2$ excess are not completely excluded, the allowed ranges are strictly bounded by the experiments above. On the other hand, if we abandon the $U(1)_{\mu-\tau}$ gauge boson as a unique resolution to the muon $g - 2$ anomaly, the available ranges of $m_{Z'}$ and $g_{Z'}$ are wide.

A detection of the light Z' gauge boson via the process $e^+e^- \rightarrow \gamma + \cancel{E}$ at the Belle II, which will record an unprecedented data sample of 50 ab^{-1} , has been recently proposed in [25, 26], where \cancel{E} is the missing energy from the $Z' \rightarrow \bar{\nu}\nu$ decays, and the Z' boson is produced through the kinetic mixing with the photon. Although the kinetic mixing is also involved in the Borexino ν - e scattering experiment, it was found that the loop-induced mixing in the $e^+e^- \rightarrow \gamma Z'$ process depends on the $q^2 = m_{Z'}^2$, whereas the mixing in the solar neutrino experiment is a constant in q^2 due to the low energy neutrinos. The kinetic

mixing parameters in the both processes are respectively written as [26]:

$$\epsilon_{\text{Belle}} = \frac{eg_{Z'}}{2\pi^2} \int_0^1 dx x(1-x) \ln \frac{m_\tau^2 - x(1-x)q^2}{m_\mu^2 - x(1-x)q^2}, \quad \epsilon_{\nu e} = \frac{eg_{Z'}}{6\pi^2} \ln \frac{m_\tau}{m_\mu}. \quad (1)$$

It is concluded that with the Belle-II integrated luminosity of 50 ab^{-1} , the significance of $e^+e^- \rightarrow \gamma + \cancel{e}$ process higher than 3σ significance can be reached, and the sensitive regions are $m_{Z'} \lesssim 1 \text{ GeV}$ and $g_{Z'} \gtrsim 0.8 \times 10^{-3}$.

If we examine the light Z' gauge boson together with the way of spontaneous $U(1)_{\mu-\tau}$ symmetry breaking, it can be found that in addition to the $m_{Z'}$ and $g_{Z'}$ parameters, the $U(1)_{\mu-\tau}$ gauge model at least needs one more new free parameter to dictate the mass of a scalar boson, in which the scalar field only carries the $U(1)_{\mu-\tau}$ charge and is responsible for the symmetry breaking. If we employ a complex singlet scalar field (ϕ_S) to accomplish the symmetry breaking, the ϕ_S - Z' - Z' coupling from the kinetic term can lead to the $\phi_S \rightarrow Z'Z'$ decay when $m_S > 2m_{Z'}$ is satisfied. If the singlet scalar can be produced with a sizable cross section, the light Z' can then be generated through the ϕ_S decay. However, if the ϕ_S field only couples to the leptons via the Yukawa interactions, we cannot have the $SU(2)_L \times U(1)_Y \times U(1)_{\mu-\tau}$ gauge invariant Yukawa couplings because the left-handed and right-handed leptons are $SU(2)_L$ doublet and singlet, respectively. Thus, it is difficult to generate the singlet scalar boson and detect the Z' signal through the $\phi_S \rightarrow Z'Z'$ channel.

We find that if two singlet vector-like leptons, which carry different $U(1)_{\mu-\tau}$ charges, are added to the model, probing the light Z' through $\phi_S \rightarrow Z'Z'$ can then be realized. The resulting model not only is $U(1)_Y$ and $U(1)_{\mu-\tau}$ gauge anomaly-free, but it also removes the scale dependence of the loop-induced kinetic mixing. As a consequence, several phenomena of interest are induced, and they include: (i) LFV branching ratios (BRs) for the $h, \phi_S \rightarrow \mu\tau$ decays can be of the order of 10^{-3} ; (ii) muon $g-2$ from the same LFV effects can achieve a level of 10^{-9} ; (iii) BR for $\tau \rightarrow \mu\phi_S^* \rightarrow \mu Z'Z'$ can be of the order of 10^{-8} ; (iv) the kinetic mixing of Eq. (1) is modified and sensitive to the relative heavy lepton masses.

With 50 ab^{-1} of data accumulated at the Belle II, the sample of τ pairs can be increased up to around 5×10^{10} , where the sensitivity to observe the LFV τ decays can reach $10^{-10} - 10^{-9}$, depending on the processes [27]. If $m_{Z'} > 2m_\mu$ and $BR(Z' \rightarrow \mu^+\mu^-) \times BR(Z' \rightarrow \bar{\nu}\nu, \mu^+\mu^+) \sim 0.2$, we will show that the BRs for the $\tau \rightarrow 3\mu + \cancel{e}$ and $\tau \rightarrow 5\mu$ decays can be $\mathcal{O}(10^{-9})$ in the extension of the SM. Intriguingly, the resulting BRs of the new tau decay channels are located in the Belle II sensitivity. Since $\tau \rightarrow 3\mu$ and $\tau \rightarrow (e, \mu)\gamma$ are

suppressed in this model, the detectable $\tau \rightarrow 3\mu + \cancel{e}$ and $\tau \rightarrow 5\mu$ decays can be used as the characteristics to distinguish from other models, which have sizable BRs for the $\tau \rightarrow 3\mu$ and $\tau \rightarrow (e, \mu)\gamma$ decays.

The uncertainty of the measured muon $g - 2$ is 0.54 ppm, where it was done by the E821 experiment at Brookhaven National Laboratory (BNL) [28], and a result over 3σ deviation from the SM prediction was obtained. The new muon $g - 2$ measurements performed by E989 experiment at Fermilab and E34 experiment at J-PARC will aim for a precision of 0.14 ppm [29] and 0.10 ppm [30], respectively. Thus, the muon $g - 2$ of $10^{-10} - 10^{-9}$ caused by the LFV effects in this model can be tested. Hence, in this work, we plan to show the impacts on the lepton-flavor conserving and LFV phenomena when the $U(1)_{\mu-\tau}$ gauge symmetry and two singlet vector-like leptons are introduced to the SM.

In the following, we start to introduce the new interactions in the extension of the SM. In a gauged $L_\mu - L_\tau$ model, we add two singlet vector-like leptons ($\ell_{4,5}$) and a complex singlet scalar field (S) into the SM, where the S field is responsible for the spontaneous $U(1)_{\mu-\tau}$ symmetry breaking, and the heavy leptons lead to the lepton-flavor changing neutral currents (LFCNCs) through the Yukawa couplings. In order to obtain the Higgs lepton-flavor violation and remove the scale dependence of the loop-induced kinetic mixing between the photon and the Z' gauge boson, the $U(1)_{\mu-\tau}$ charges of ℓ_4 and ℓ_5 must be opposite in sign. When the charge of ℓ_4 is determined, the charge of S then is certain. For clarity, we show the $U(1)_{\mu-\tau}$ charges of the leptons and S field in Table I. Accordingly, the Yukawa interactions, which satisfy the $SU(2)_L \times U(1)_Y \times U(1)_{\mu-\tau}$ gauge symmetry, are written as:

$$\begin{aligned}
-\mathcal{L}_Y = & Y_\ell \bar{L}_\ell H \ell_R + y_\mu \bar{L}_\mu H \ell_{4R} + y_\tau \bar{\ell}_{4L} \tau_R S + m_{4L} \bar{\ell}_{4L} \ell_{4R} \\
& + y'_\tau \bar{L}_\tau H \ell_{5R} + y'_\mu \bar{\ell}_{5L} \mu_R S^\dagger + y_S \bar{\ell}_4 \ell_5 S + m_{5L} \bar{\ell}_{5L} \ell_{5R} + H.c. ,
\end{aligned} \tag{2}$$

where L_ℓ denotes the SM doublet lepton, $f_{R(L)} = P_{R(L)} f$ with $P_{R(L)} = (1 \pm \gamma_5)/2$, H is the SM Higgs doublet, and $m_{4L,5L}$ are the heavy lepton masses. The electroweak and $U(1)_{\mu-\tau}$ symmetries can be spontaneously broken through $\langle H \rangle = (v+h)/\sqrt{2}$ and $\langle S \rangle = (v_S + \phi_S)/\sqrt{2}$, where $v(v_S)$ is the vacuum expectation value (VEV) of the $H(S)$ field. From Eq. (2), it can be seen that ℓ_4 and ℓ_5 can mix together through the $y_S \bar{\ell}_4 \ell_5 S$ term when the $U(1)_{\mu-\tau}$ symmetry is broken. In order to simplify the following formulation, we take the basis, $\ell'_4 = \cos \alpha \ell_4 - \sin \alpha \ell_5$ and $\ell'_5 = \sin \alpha \ell_4 + \cos \alpha \ell_5$, so that the 2×2 mass matrix of ℓ_4 and ℓ_5

TABLE I: $U(1)_{\mu-\tau}$ charges of leptons and S field.

	e	μ	τ	ℓ_4	ℓ_5	S
$U(1)$	0	1	-1	1	-1	2

is diagonalized as:

$$\begin{pmatrix} m_{L1} & 0 \\ 0 & m_{L2} \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} m_{4L} & \frac{y_S v_S}{\sqrt{2}} \\ \frac{y_S v_S}{\sqrt{2}} & m_{5L} \end{pmatrix} \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}, \quad (3)$$

where the m_{L1} , m_{L2} , and mixing angle α can be related to the $m_{4L,5L}$ and $y_S v_S$ parameters as:

$$m_{L1,L2} = \frac{1}{2} \left(m_{4L} + m_{5L} \pm \sqrt{(m_{5L} - m_{4L})^2 + 2y_S^2 v_S^2} \right),$$

$$\tan 2\alpha = \frac{\sqrt{2} y_S v_S}{m_{5L} - m_{4L}}. \quad (4)$$

We note that in general, the SM Higgs can mix with the scalar ϕ_S via the scalar potential. Since the mixing is a new free parameter, in order to avoid the constraint from the precision Higgs measurements, hereafter, we set the mixing to be small and neglect its contributions.

In order to obtain the Z' mass and the S gauge coupling to the Z' boson, we write the covariant derivative of the S field to be $D_\mu = \partial_\mu + i g_{Z'} X_S Z'_\mu$, where $X_S = 2$ is the $U(1)_{\mu-\tau}$ charge of the S field. With $\langle S \rangle = (v_S + \phi_S)/\sqrt{2}$, the Z' mass and $\phi_S - Z' - Z'$ coupling can be obtained through the kinetic term as:

$$(D^\mu S)^\dagger (D_\mu S) \supset \frac{1}{2} (v_S + \phi_S)^2 g_{Z'}^2 X_S^2 Z'_\mu Z'^\mu,$$

$$m_{Z'} = 2g_{Z'} v_S, \quad \phi_S - Z' - Z' : \frac{2m_{Z'}^2}{v_S} g_{\mu\nu}. \quad (5)$$

After electroweak and $U(1)_{\mu-\tau}$ symmetry breaking, the lepton-flavor mixing information can be obtained from Eq. (2). If we combine the SM leptons and the heavy leptons to form a multiplet state in flavor space, denoted by $\ell'^T = (\boldsymbol{\ell}, \boldsymbol{\Psi}_\ell)$ with $\boldsymbol{\ell} = (e, \mu, \tau)$ and $\boldsymbol{\Psi}_\ell^T = (\ell'_4, \ell'_5)$, the 5×5 lepton mass matrix can be written as:

$$\bar{\ell}'_L M_{\ell'} \ell'_R = \left(\bar{\boldsymbol{\ell}}_L, \bar{\boldsymbol{\Psi}}_{\ell L} \right) \left(\begin{array}{c|c} \boldsymbol{m}_{\ell 3 \times 3} & \boldsymbol{\delta m}_1 \\ \hline \boldsymbol{\delta m}_2^T & \boldsymbol{m}_L \end{array} \right)_{5 \times 5} \begin{pmatrix} \boldsymbol{\ell}_R \\ \boldsymbol{\Psi}_{\ell R} \end{pmatrix}, \quad (6)$$

where $\text{diag}\mathbf{m}_\ell = (m_e, m_\mu, m_\tau)$, $m_f = vY_f/\sqrt{2}$, $\text{diag}\mathbf{m}_L = (m_{L1}, m_{L2})$, and $\delta\mathbf{m}_{1,2}$ are given by:

$$\delta\mathbf{m}_1^T = \begin{pmatrix} 0, & \frac{vy_\mu}{\sqrt{2}}c_\alpha, & -\frac{vy'_\tau}{\sqrt{2}}s_\alpha \\ 0, & \frac{vy_\mu}{\sqrt{2}}s_\alpha, & \frac{vy'_\tau}{\sqrt{2}}c_\alpha \end{pmatrix}, \quad \delta\mathbf{m}_2^T = \begin{pmatrix} 0, & -\frac{vSy'_\mu}{\sqrt{2}}s_\alpha, & \frac{vSy_\tau}{\sqrt{2}}c_\alpha \\ 0, & \frac{vSy'_\mu}{\sqrt{2}}c_\alpha, & \frac{vSy_\tau}{\sqrt{2}}s_\alpha \end{pmatrix} \quad (7)$$

with $c_\alpha = \cos \alpha$ and $s_\alpha = \sin \alpha$. To diagonalize the mass matrix $M_{\ell'}$ in Eq. (6), we introduce two unitary matrices $V_{R,L}$. Since we take the flavor mixing effects are perturbative and are suppressed by $m_{L1,L2}$, the 5×5 flavor mixing matrices can be simplified as:

$$V_\chi \approx \left(\begin{array}{c|c} \mathbb{1}_{3 \times 3} & -\epsilon_\chi \\ \hline \epsilon_\chi^\dagger & \mathbb{1}_{2 \times 2} \end{array} \right)_{5 \times 5}, \quad (8)$$

where we only retain the leading contributions, and the effects, which are smaller than ϵ_χ with $\chi = R, L$, have been dropped, such as $\epsilon_\chi^\dagger \epsilon_\chi$, $\mathbf{m}_\ell \delta\mathbf{m}_{1,2}/\mathbf{m}_L^2$, etc. The explicit expressions of ϵ_χ then are given by:

$$\epsilon_L^\dagger = \begin{pmatrix} 0, & \frac{vy_\mu}{\sqrt{2}m_{L1}}c_\alpha, & -\frac{vy'_\tau}{\sqrt{2}m_{L1}}s_\alpha \\ 0, & \frac{vy_\mu}{\sqrt{2}m_{L2}}s_\alpha, & \frac{vy'_\tau}{\sqrt{2}m_{L2}}c_\alpha \end{pmatrix}, \quad \epsilon_R^\dagger = \begin{pmatrix} 0, & -\frac{vSy'_\mu}{\sqrt{2}m_{L1}}s_\alpha, & \frac{vSy_\tau}{\sqrt{2}m_{L1}}c_\alpha \\ 0, & \frac{vSy'_\mu}{\sqrt{2}m_{L2}}c_\alpha, & \frac{vSy_\tau}{\sqrt{2}m_{L2}}s_\alpha \end{pmatrix} \quad (9)$$

where the Yukawa couplings $y_{\mu,\tau}$ and $y'_{\mu,\tau}$ are taken as real numbers.

After rotating the lepton weak states to physical states based on the V_R and V_L , the Yukawa couplings of the SM Higgs and ϕ_S to the charged leptons from Eq. (2) are expressed as:

$$\begin{aligned} -\mathcal{L}_{h,\phi_S} = & \left(\bar{\ell}_L, \bar{\Psi}_{\tau'L} \right) V_L \left(\begin{array}{c|c} \mathbf{m}_{\ell 3 \times 3} & \delta\mathbf{m}_1 \\ \hline 0 & 0 \end{array} \right) V_R^\dagger \begin{pmatrix} \ell_R \\ \Psi_{\tau'R} \end{pmatrix} \frac{h}{v} \\ & + \left(\bar{\ell}_L, \bar{\Psi}_{\tau'L} \right) V_L \left(\begin{array}{c|c} 0 & 0 \\ \hline \delta\mathbf{m}_2^T & 0 \end{array} \right) V_R^\dagger \begin{pmatrix} \ell_R \\ \Psi_{\tau'R} \end{pmatrix} \frac{\phi_S}{v_S}, \end{aligned} \quad (10)$$

where we still use ℓ to represent the light leptons, however, for the mass eigenstate of the heavy lepton, we use $\Psi_{\tau'}^T = (\tau', \tau'')$ instead of $\Psi_{\ell'}^T = (\ell'_4, \ell'_5)$. With the leading expansions in Eq. (8), the Higgs and ϕ_S Yukawa couplings to the light charged leptons can be summarized as:

$$\begin{aligned} -\mathcal{L}_Y^{h,\phi_S} \supset & \frac{m_\ell}{v} \bar{\ell}_L \ell_R h - \frac{y_\mu y_\tau}{2} \left(\frac{c_\alpha^2}{m_{L1}} + \frac{s_\alpha^2}{m_{L2}} \right) \bar{\mu}_L \tau_R (v_S h + v \phi_S) \\ & - \frac{y'_\mu y'_\tau}{2} \left(\frac{s_\alpha^2}{m_{L1}} + \frac{c_\alpha^2}{m_{L2}} \right) \bar{\tau}_L \mu_R (v_S h + v \phi_S) + H.c. \end{aligned} \quad (11)$$

The full Yukawa couplings are shown in the appendix; here we just show the relevant parts. From Eq. (35), it can be seen that the modified Higgs couplings to μ - and τ -lepton are proportional to s_α and $m_{L2} - m_{L1}$; thus, the modifications can be suppressed by a small s_α or/and $m_{L2} \approx m_{L1}$. Since the mixing between τ' and τ'' does not influence the LFV effects, in order to simplify the analysis, hereafter we take $s_\alpha = 0$ and $c_\alpha = 1$. According to Eq. (10), in addition to the $h \rightarrow \mu\tau$ and $\tau \rightarrow \mu Z' Z'$ decays, the h - and ϕ_S -mediated LFV effects can also lead to a sizable muon $g - 2$. Since the LFV currents involve $\bar{\tau}_L \mu_R$ and $\bar{\mu}_L \tau_R$, the muon $g - 2$ can be enhanced by m_τ due to the τ chirality flip. Although the $\tau \rightarrow 3\mu$ decay is allowed in the model, since the vertices are suppressed by the m_μ/v and have no other enhancing factor, the resulting branching ratio is $\sim 8 \times 10^{-13}$ and is far below the current experimental upper bound of 2.1×10^{-8} [34]. Similarly, the BR for the $\tau \rightarrow \mu\gamma$ decay is also small. The LFV couplings, which involve heavy leptons, can be found in the appendix.

Next, we discuss the influence of the vector-like leptons on the Z and Z' gauge couplings to the leptons. Since the introduced heavy leptons are $SU(2)_L$ singlets and carry the hypercharge $Y = -1$, which is the same as that carried by the right-handed light leptons. Therefore, the Z gauge couplings to the right-handed leptons are flavor conserving at the tree level. However, because the left-handed light and heavy leptons carry different $U(1)_Y$ charges, the Z -mediated LFCNCs at the tree level occur in the left-handed leptons. To show the modified and new Z gauge couplings to the leptons, we write the interactions in the physical lepton states as:

$$\mathcal{L}_Z = -(\bar{\ell}_L, \bar{\Psi}_{\tau'L}) \gamma_\mu V_L \left(\begin{array}{c|c} C_L^\ell \mathbb{1}_{3 \times 3} & 0 \\ \hline 0 & C_R^\ell \mathbb{1}_{2 \times 2} \end{array} \right) V_L^\dagger \begin{pmatrix} \ell_L \\ \Psi_{\tau'L} \end{pmatrix} Z^\mu, \quad (12)$$

$$\approx -(\bar{\ell}_L, \bar{\Psi}_{\tau'L}) \gamma_\mu \left(\begin{array}{c|c} C_L^\ell \mathbb{1}_{3 \times 3} & (C_L^\ell - C_R^\ell) \epsilon_L \\ \hline (C_L^\ell - C_R^\ell) \epsilon_L^\dagger & C_R^\ell \mathbb{1}_{2 \times 2} \end{array} \right) \begin{pmatrix} \ell_L \\ \Psi_{\tau'L} \end{pmatrix} Z^\mu, \quad (13)$$

$$C_L^\ell = \frac{g}{2c_W} (2s_W^2 - 1), \quad C_R^\ell = \frac{gs_W^2}{c_W}, \quad (14)$$

where we have dropped the $\epsilon_L \epsilon_L^\dagger$ and $\epsilon_L^\dagger \epsilon_L$ effects; $c_W = \cos \theta_W$, $s_W = \sin \theta_W$, and θ_W is the Weinberg's angle. Based on this approximation, the Z -boson couplings to the light charged leptons are still flavor conserving. Nevertheless, the LFV processes can occur in the heavy leptons to the light leptons. Hence, with the leading order approximation to the flavor

mixing matrices, the Z -boson couplings to the charged leptons can be simplified as:

$$-\mathcal{L}_Z \approx [C_L^\ell \bar{\ell}_L \gamma_\mu \ell_L + C_R^\ell \bar{\ell}_R \gamma_\mu \ell_R + C_R^\ell \bar{\Psi}_{\tau'} \gamma_\mu \Psi_{\tau'}] Z^\mu - \frac{g}{2c_W} \left(\frac{vy_\mu}{\sqrt{2}m_{L1}} \bar{\mu}_L \gamma_\mu \tau'_L + \frac{vy'_\tau}{\sqrt{2}m_{L2}} \bar{\tau}_L \gamma_\mu \tau''_L + H.c. \right) Z^\mu. \quad (15)$$

Although the introduced $\ell_{4,5}$ leptons are vectorial couplings to the Z' boson, since the charged leptons carry different $U(1)_{\mu-\tau}$ charges, and the flavor mixing matrices in general distinguish the lepton chirality, we thus write the Z' couplings to the leptons as:

$$\mathcal{L}_{Z'} = -g_{Z'} \bar{\ell}' \gamma^\mu V_R Q' V_R^\dagger P_R \ell' Z'_\mu - g_{Z'} \bar{\ell}' \gamma^\mu V_L Q' V_L^\dagger P_L \ell' Z'_\mu - g_{Z'} \bar{\nu}_\ell \gamma^\mu Q P_L \nu_\ell Z'_\mu, \quad (16)$$

where $\ell'^T = (e, \mu, \tau, \tau', \tau'')$, Q' and Q denotes the $U(1)'$ charges of the charged leptons and neutrinos, and are expressed by $\text{dia}Q' = (0, 1, -1, 1, -1)$ and $\text{dia}Q = (0, 1, -1)$, respectively. Taking the leading approximation, the couplings to the charged leptons can be simplified as:

$$-\mathcal{L}_{Z'} \approx g_{Z'} \left(\bar{\ell}, \bar{\Psi}_{\tau'} \right) \gamma^\mu Q' \begin{pmatrix} \ell \\ \Psi_{\tau'} \end{pmatrix} Z'_\mu - g_{Z'} \frac{2v_S y_\tau}{\sqrt{2}m_{L1}} \bar{\tau}_R \gamma^\mu \tau'_R Z'_\mu + g_{Z'} \frac{2v_S y'_\mu}{\sqrt{2}m_{L2}} \bar{\mu}_R \gamma^\mu \tau''_R Z'_\mu + H.c. \quad (17)$$

It can be seen that the Z' couplings to the lepton pairs are still flavor conserved. The Z' mediated flavor changing effects only occur in the right-handed currents and at the $\tau' - \tau$ and $\tau'' - \mu$ vertices. Since we focus on the light Z' and small $g_{Z'}$, these flavor-changing couplings cannot have a significance influence on the muon $g - 2$. In addition, the contributions to the τ' and τ'' decay widths are also small.

Based on the introduced interactions, in the following, we discuss the relevant phenomena of interest. From Eq. (11), it is found that the LFV coupling $\mu - \tau - h$ leads to the $h \rightarrow \mu\tau$ decay, and the associated BR can be written as:

$$BR(h \rightarrow \mu\tau) = \frac{v_S^2 (|a_L|^2 + |a_R|^2)}{8\pi\Gamma_h} m_h, \quad (18)$$

$$a_L = \frac{y_\mu y_\tau}{2m_{L1}}, \quad a_R = \frac{y'_\mu y'_\tau}{2m_{L2}},$$

where $\tau\mu$ indicates the sum of $\bar{\mu}\tau + \bar{\tau}\mu$, and Γ_h is the Higgs width. With $m_h = 125$ GeV, $\Gamma_h \approx 4.21$ MeV, the parameters can be reformulated as:

$$\sqrt{|a_L|^2 + |a_R|^2} \approx \frac{2.67 \times 10^{-3}}{v_S} \sqrt{\frac{BR(h \rightarrow \tau\mu)}{0.84 \times 10^{-2}}}, \quad (19)$$

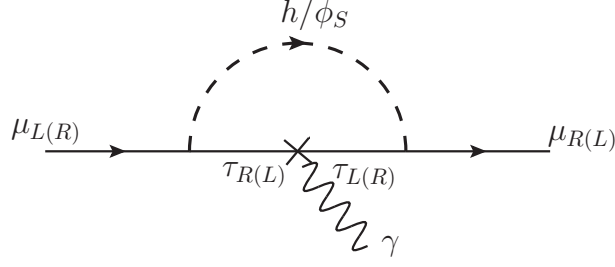


FIG. 1: Sketched Feynman diagram for the Higgs- and ϕ_S -mediated muon $g - 2$.

where $BR(h \rightarrow \mu\tau)$ can be taken from the experimental data, and the current upper limits from ATLAS and CMS are 1.43% [33] and 1.26% [31, 32], respectively. Moreover, the LFV effects in Eq. (11) can also contribute to the muon $g - 2$, and the Feynman diagram is sketched in Fig. 1. Accordingly, the h - and ϕ_S -mediated muon $g - 2$ can be derived as:

$$\Delta a_\mu = -Q_\tau m_\mu m_\tau \frac{a_R a_L}{4\pi^2} \left[\frac{v_S^2}{m_h^2} \left(\ln \frac{m_h^2}{m_\tau^2} - \frac{3}{2} \right) + \frac{v^2}{m_S^2} \left(\ln \frac{m_S^2}{m_\tau^2} - \frac{3}{2} \right) \right], \quad (20)$$

where $Q_\tau = -1$ is the τ -lepton electric charge. From Eqs. (18) and (20), it can be seen that there is a strong correlation between $BR(h \rightarrow \mu\tau)$ and Δa_μ . We show the contours for the $BR(h \rightarrow \mu\tau)$ in units of 10^{-2} (dashed) and Δa_μ in units of 10^{-10} (solid) as a function of a_R and a_L in Fig. 2(a), where $v_S = 120$ GeV and $m_S = 10$ GeV are used. From the plot, it can be found that a large $BR(h \rightarrow \mu\tau)$ and Δa_μ of 10×10^{-10} can be reconciled by the scalar-mediated LFV effects.

Basically, a_R and a_L appearing in Eqs. (18) and (20) can be taken as two independent parameters; however, if we adopt the scheme with $a_L \approx a_R$, Δa_μ then can be expressed in terms of $BR(h \rightarrow \mu\tau)$ as:

$$\frac{\Delta a_\mu}{BR(h \rightarrow \mu\tau)} \approx \frac{m_\mu m_\tau \Gamma_h}{\pi m_h^3} \left[\left(\ln \frac{m_h^2}{m_\tau^2} - \frac{3}{2} \right) + \frac{v^2}{(m_S v_S)^2} \left(\ln \frac{m_S^2}{m_\tau^2} - \frac{3}{2} \right) \right]. \quad (21)$$

Since there is no other free parameter in the first term of Eq. (21), if we take $BR(h \rightarrow \mu\tau) \sim 0.84 \times 10^{-2}$, the Higgs contribution can be estimated to be $\Delta a_\mu^h \sim 7.5 \times 10^{-12}$, which is far below the current experimental value. Hence, the dominant contribution to Δa_μ is from the ϕ_S mediation. Based on Eq. (21), we show the contours for Δa_μ (in units of 10^{-10}) as a function of $BR(h \rightarrow \mu\tau)$ and v_S with $m_S = 10$ GeV in Fig. 2(b). From the plot, it can be seen that when the value of $BR(h \rightarrow \mu\tau)$ is around 0.2×10^{-2} , the value of Δa_μ can still reach 10×10^{-10} , where v_S is around 70 GeV and $m_{Z'} \approx 0.14$ GeV for $g_{Z'} = 10^{-3}$. In order to

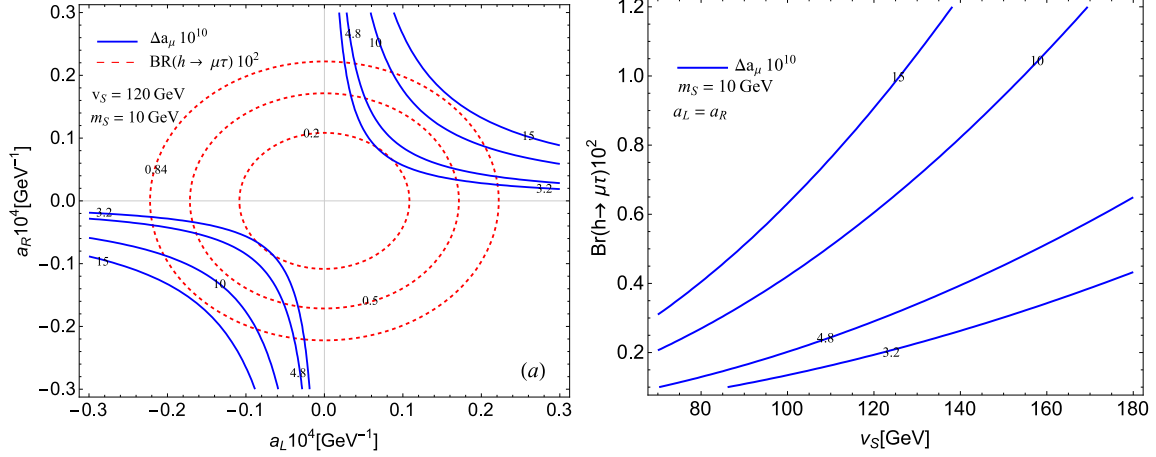


FIG. 2: (a) Contours for $BR(h \rightarrow \mu\tau)$ in units of 10^{-2} (dashed) and Δa_μ in units of 10^{-10} (solid) as a function of a_R and a_L , where $v_S = 120 \text{ GeV}$ and $m_S = 10 \text{ GeV}$ are taken. (b) Contours for Δa_μ (in units of 10^{-10}) with $a_L = a_R$ as a function of $BR(h \rightarrow \mu\tau)$ and v_S , where $m_S = 10 \text{ GeV}$ is used.

clearly see the impact of the LFV effect on the muon $g-2$, we show the contours for Δa_μ as a function of $m_{Z'}$ and $g_{Z'}$ in Fig. 3, where the results bounded by the dot-dashed, dashed, and solid lines represent the contributions from the Z' boson, ϕ_S , and $Z' + \phi_S$, respectively; the taken region for each contribution is given by $\Delta a_\mu^{Z'} = (2, 8) \times 10^{-10}$, $\Delta a_\mu^{\phi_S} = (1, 10) \times 10^{-10}$, and $\Delta a_\mu^{Z'+\phi_S} = (12.7, 44.7) \times 10^{-10}$; and the Z' contribution is written as [26]:

$$\Delta a_\mu^{Z'} = \frac{g_{Z'}^2}{8\pi^2} \int_0^1 dx \frac{2m_\mu^2 x^2 (1-x)}{x^2 m_\mu^2 + (1-x)m_{Z'}^2}. \quad (22)$$

According to the figure, when $\Delta a_\mu^{Z'}$ is less than 10×10^{-10} at $g_{Z'} \sim \mathcal{O}(10^{-3})$, due to the ϕ_S contribution, the muon $g-2$ can reach the current data with 2σ errors.

As shown above, in order to enhance the muon $g-2$ up to the $10^{-10} - 10^{-9}$ level, it prefers a light ϕ_S . Under this circumstance, the ϕ_S predominantly decays into $\tau\mu$ and $Z'Z'$. According to the gauge coupling in Eq. (5) and the Yukawa couplings in Eq. (11), the ϕ_S partial decay rates can be expressed as:

$$\begin{aligned} \Gamma(\phi_S \rightarrow Z'Z') &= \frac{m_S^3}{16\pi v_S^2}, \\ \Gamma(\phi_S \rightarrow \tau\mu) &= \frac{v^2 (|a_L|^2 + |a_R|^2)}{8\pi} m_S, \\ &= 0.28 \times 10^{-6} \frac{m_S v^2}{v_S^2} \frac{BR(h \rightarrow \tau\mu)}{0.84 \times 10^{-2}}, \end{aligned} \quad (23)$$

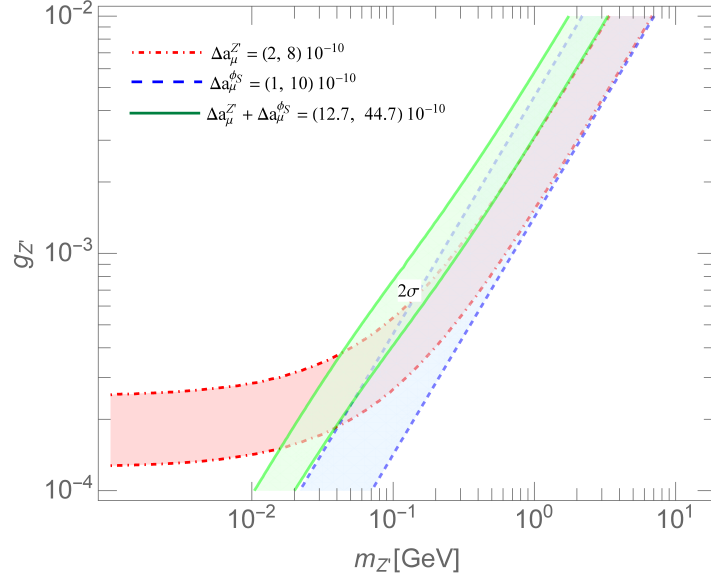


FIG. 3: Contours for Δa_μ as a function of $m_{Z'}$ and $g_{Z'}$, where the results bounded by the dot-dashed, dashed, and solid lines denote the Z' , ϕ_S , and $Z' + \phi_S$ contributions, respectively.

where the lepton and Z' mass effects are neglected, and $\tau\mu$ indicates the sum of $\bar{\tau}\mu$ and $\bar{\mu}\tau$ channels. In the last line, we have applied the result of Eq. (19). It can be clearly seen that $\Gamma(\phi_S \rightarrow \tau\mu) \ll \Gamma(\phi_S \rightarrow Z'Z')$. Accordingly, the BRs for the ϕ_S decays are shown as:

$$BR(\phi_S \rightarrow Z'Z') \approx 1, \quad BR(\phi_S \rightarrow \tau\mu) \approx 2.16 \times 10^{-3} \left(\frac{20 \text{ GeV}}{m_S} \right)^2 \frac{BR(h \rightarrow \mu\tau)}{0.84 \times 10^{-2}}. \quad (24)$$

In addition to the $\phi_S \rightarrow \tau\mu$ decay, the same LFV effects can lead to $\tau \rightarrow \mu Z'Z'$ decay through the ϕ_S mediation. The differential branching fraction as a function of $Z'Z'$ invariant mass is shown as:

$$\begin{aligned} \frac{dBR(\tau \rightarrow \mu Z'Z')}{dq^2} &\approx \frac{m_\tau}{64\pi^2 m_h} \frac{\Gamma_h}{\Gamma_\tau} BR(h \rightarrow \mu\tau) \\ &\times \frac{(q^2 - 2m_{Z'}^2)^2 + 8m_{Z'}^4}{v_S^4 m_S^2} \left(1 - \frac{q^2}{m_\tau^2} \right)^2 \sqrt{1 - \frac{4m_{Z'}^2}{q^2}}. \end{aligned} \quad (25)$$

Based on the result, we show the contours for $BR(\tau \rightarrow \mu Z'Z')$ (dot-dashed) as a function of $BR(h \rightarrow \mu\tau)$ and v_S in Fig. 4, where $m_S = 10 \text{ GeV}$ is used; for comparison, we also show the muon $g - 2$ (solid) in the same plot, and the numbers on the contour lines denote the values of $BR(\tau \rightarrow \mu Z'Z')$ and Δa_μ , which have been rescaled by a 10^{-8} and a 10^{-10}

factor, respectively. From the plot, it can be seen that when Δa_μ is of the order of 10^{-10} , the associated $BR(\tau \rightarrow \mu Z' Z')$ is in the order of 10^{-8} .

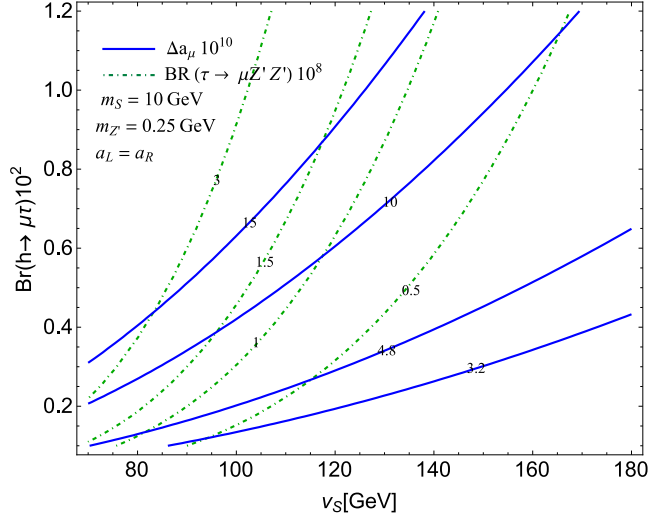


FIG. 4: Contours for $BR(\tau \rightarrow \mu Z' Z')$ in units of 10^{-8} (dot-dashed) and Δa_μ in units of 10^{-10} (solid) as a function of $BR(h \rightarrow \mu\tau)$ and v_S , where $m_S = 10$ GeV, $m_{Z'} = 0.25$ GeV, and $a_L = a_R$ are taken.

The possible detecting signals for $\tau \rightarrow \mu Z' Z'$ depend on the Z' mass. If $m_{Z'} < 2m_\mu$, the Z' gauge boson can only decay into ν_μ and ν_τ pairs. Thus, the detecting signals will be $\tau \rightarrow \mu + \cancel{E}$ with \cancel{E} being a missing energy. Since the $\tau \rightarrow \mu \bar{\nu}_\nu \nu_\tau$ process of $BR \approx 17.27\%$ in the SM is the main background, the small $BR(\tau \rightarrow \mu Z' Z')$ cannot be distinguished from the errors of $BR^{\text{exp}}(\tau \rightarrow \mu \bar{\nu}_\nu \nu_\tau) = (17.39 \pm 0.04)\%$ [34]. In this case, we cannot see the signal for the $\tau \rightarrow \mu Z' Z'$ decay. However, when $m_{Z'} > 2m_\mu$, in addition to the neutrino pair, the Z' can also decay into muon pair. Therefore, the signals can be $\tau \rightarrow 3\mu \bar{\nu} \nu$ and $\tau \rightarrow 5\mu$, where ν includes the ν_μ and ν_τ neutrinos, and the decay chains are shown as:

$$\tau \rightarrow \mu Z' Z'; Z' \rightarrow \bar{\nu} \nu, Z' \rightarrow \mu^+ \mu^- \quad (26)$$

$$\tau \rightarrow \mu Z' Z'; Z' \rightarrow \mu^+ \mu^-, Z' \rightarrow \mu^+ \mu^-. \quad (27)$$

The BRs for the $Z' \rightarrow (\bar{\nu} \nu, \mu^+ \mu^-)$ decays can be simply formulated as:

$$BR(Z' \rightarrow \bar{\nu} \nu) = \frac{1}{1 + M_\mu^2}, \quad BR(Z' \rightarrow \mu^+ \mu^-) = \frac{M_\mu^2}{1 + M_\mu^2}, \quad (28)$$

$$M_\mu^2 = \left(1 + \frac{2m_\mu^2}{m_{Z'}^2}\right) \sqrt{1 - \frac{4m_\mu^2}{m_{Z'}^2}}, \quad (29)$$

where the BRs only depend on the $m_{Z'}$ parameter. For illustration, we show the values of BRs with respect to some selected $m_{Z'}$ values in Table II. According to the results in Fig. 4 and the values in Table II, it can be found that the BRs for $\tau \rightarrow (3\mu + \cancel{E}, 5\mu)$ can reach a level of 10^{-9} , which is the detecting sensitivity at the Belle II.

TABLE II: Values of branching ratios for the $Z' \rightarrow (\bar{\nu}\nu, \mu^+\mu^-)$ decays with respect to the selected $m_{Z'}$ values, where ν includes the ν_μ and ν_τ neutrinos.

$m_{Z'}$ [GeV]	0.22	0.25	0.3	0.34	0.38
$BR(Z' \rightarrow \bar{\nu}\nu)$	0.71	0.58	0.53	0.52	0.51
$BR(Z' \rightarrow \mu^+\mu^-)$	0.29	0.42	0.47	0.48	0.49

Finally, we examine the influence of vector-like leptons on the $e^+e^- \rightarrow \gamma Z'$ process, which is arisen from the kinetic mixing [26]. The Feynman diagram for the loop-induced kinetic mixing is sketched in Fig. (6), where the leptons inside the loop include $\ell' = \mu, \tau, \tau'$, and τ'' . Accordingly, the effective Lagrangian is expressed as:

$$\begin{aligned}\mathcal{L}_{\text{mix}} &= -\frac{\epsilon}{2}F_{\mu\nu}Z'^{\mu\nu}, \\ &= -\Pi(q^2) \left(q^2 \epsilon_\gamma \cdot \epsilon_{Z'}^* - q \cdot \epsilon_\gamma q \cdot \epsilon_{Z'}^* \right),\end{aligned}\quad (30)$$

where $F_{\mu\nu}$ and $Z'_{\mu\nu}$ are the $U(1)_{\text{em}}$ and $U(1)_{\mu-\tau}$ gauge field strength tensors, respectively, and the $\epsilon = \Pi(q^2)$ can be derived as:

$$\Pi(q^2) = \frac{8eg_{Z'}}{(4\pi)^2} \int_0^1 dx x(1-x) \left[\ln \frac{m_\tau^2 - x(1-x)q^2}{m_\mu^2 - x(1-x)q^2} + \ln \frac{m_{L2}^2 - x(1-x)q^2}{m_{L1}^2 - x(1-x)q^2} \right]. \quad (31)$$

Since the $U(1)_{\mu-\tau}$ charges of ℓ_4 and ℓ_5 are opposite in sign, the scale dependent factor from a renormalization scheme is cancelled, and the contributions of the vector-like leptons are basically similar to those of μ and τ leptons. In order to present the influence of τ' and τ'' , we show the ϵ as a function of E_γ in Fig. 6, where the relation of E_γ and q^2 is given by $E_\gamma = (s - q^2)/(2\sqrt{s})$, \sqrt{s} is the center-of-mass energy of e^+e^- , and $\sqrt{s} = 10.58$ GeV is used. In the left panel, the solid, dashed, dotted, and dot-dashed lines denote the results of $m_{L2} = (0.7, 0.9, 1.1, 1.5)$ TeV, respectively, and $m_{L1} = 0.7$ TeV is fixed. The horizontal lines denote the same situations for the $\epsilon_{\nu e}$ results. In the right panel, we fix $m_{L2} = 0.7$ TeV and show the results with $m_{L1} = (0.7, 0.9, 1.1, 1.5)$ TeV for ϵ and $\epsilon_{\nu e}$. We note that the results with $m_{L1} = m_{L2} = 0.7$ TeV (solid) are the same as those without vector-like

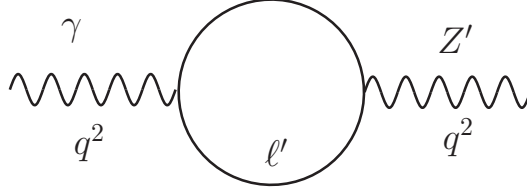


FIG. 5: Sketched Feynman diagram for the kinetic mixing of γ and Z' , where the leptons inside the loop include $\ell' = \mu, \tau, \tau'$, and τ'' .

leptons. From the plots, it can be seen that ϵ in the small E_γ region (i.e., larger $m_{Z'}$) is sensitive to the ratio m_{L2}/m_{L1} . However, for the $e^+e^- \rightarrow \gamma + \cancel{e}$ process, the SM backgrounds dominate in the small E_γ region. To clearly understand how the $q^2 = m_{Z'}^2$, (m_{L1} , and m_{L2}) affect the discovery significance, with the selected values of $\sqrt{q^2} = m_{Z'}$ and (m_{L1}, m_{L2}), we show the numerical values for the signal (N_S) and background (N_B) numbers and the corresponding significance, defined by $N_S/\sqrt{N_B + N_S}$, in Table III, where $g_{Z'} = 10^{-3}$ is fixed and the integrated luminosity of 50 ab^{-1} is used in the numerical calculations; here we applied formulas for the signal and SM background cross section in Ref. [26]. It can be found that the significance can be over 3σ as $m_{Z'} \lesssim 1.0 \text{ GeV}$ and is increased(decreased) for $m_{L1} < (>) m_{L2}$.

TABLE III: Number of signal events N_S and corresponding significance, defined by $N_S/\sqrt{N_S + N_B}$, for several taken values of (m_{L1}, m_{L2}), where $g_{Z'} = 10^{-3}$ is fixed, the integrated luminosity of 50 ab^{-1} is used, and the cases for $\sqrt{q^2} = m_{Z'} = (1.0, 1.5) \text{ GeV}$ are presented. The number of background events is $N_B = 8(30)$ for $m_{Z'} = 1.0(1.5) \text{ GeV}$.

$(m_{L1}, m_{L2}) \text{ [TeV]}$	(0.7, 0.7)	(0.7, 1.1)	(0.7, 1.5)	(1.1, 0.7)	(1.5, 0.7)
N_S	14 (11)	19 (14)	23 (17)	11 (9)	9 (8)
Significance	3.0 (1.7)	3.6 (2.1)	4.1 (2.5)	2.5 (1.4)	2.2 (1.3)

In summary, we studied the $U(1)_{L_\mu - L_\tau}$ extension of the SM by including a pair of singlet vector-like leptons, where both heavy leptons carry different $L_\mu - L_\tau$ charges. We employ a complex singlet scalar field to dictate the spontaneous $U(1)_{L_\mu - L_\tau}$ symmetry breaking. With $g_{Z'} \sim O(10^{-3})$, the VEV of the singlet scalar field must be at the electroweak scale in order to obtain $m_{Z'}$ in the MeV to GeV region. It is found that the muon $g - 2$ and the $h \rightarrow \mu\tau$ decay are strongly correlated in this model; when $BR(h \rightarrow \nu\tau) \sim 10^{-3}$, the

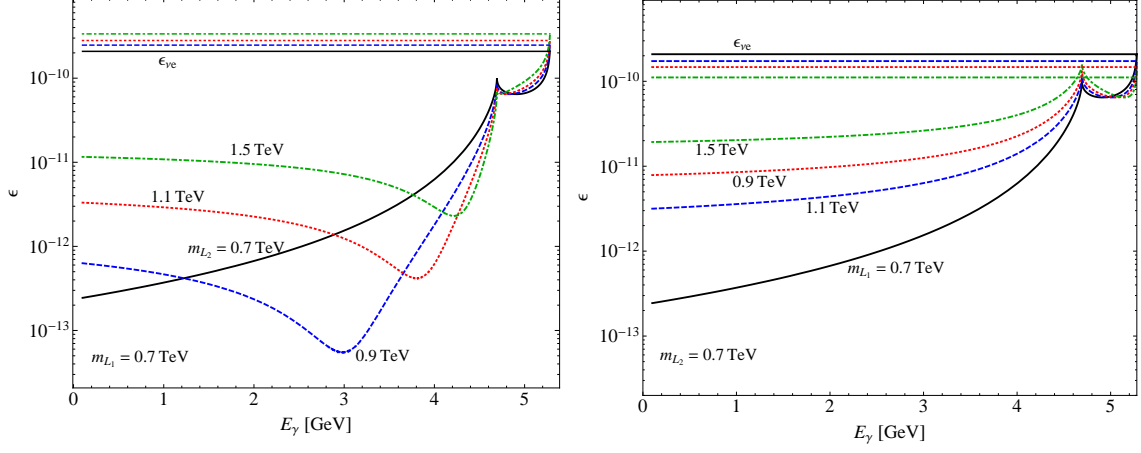


FIG. 6: Left panel: ϵ for $m_{L2} > m_{L1}$ in Eq. (31) as a function of E_γ , where $E_\gamma = (s - q^2)/(2\sqrt{s})$, and $\sqrt{s} = 10.58$ GeV and $m_{L1} = 0.7$ TeV are used; the solid, dashed, dotted, and dot-dashed lines denote the results of $m_{L2} = (0.7, 0.9, 1.1, 1.5)$ TeV, respectively. The horizontal lines are the same situations but for the case of $\epsilon_{\nu e} = \Pi(0)$. The contributions of $m_{L1} = m_{L2} = 0.7$ TeV are the same as those without vector-like leptons. Right panel: legend is the same as that in the left panel, but for $m_{L1} > m_{L2}$.

scalar-mediated muon $g - 2$ can reach 10×10^{-10} . Moreover, even with $g_{Z'} \sim 10^{-4}$, the muon $g - 2$ can fit the current data with 2σ errors. The kinetic mixing in the $e^+e^- \rightarrow \gamma + \cancel{e}$ process not only depends on the $q^2 = m_{Z'}^2$, but also is sensitive to the ratio of m_{L2}/m_{L1} ; as a result, the significance to discover the signal of $e^+e^- \rightarrow \gamma \cancel{e}$ is increasing (decreasing) for $m_{L2} > (<) m_{L1}$. In addition, it is found that $BR(\tau \rightarrow \mu Z' Z')$ by the ϕ_S mediation can be of the $\mathcal{O}(10^{-8})$. When the BRs for $Z' \rightarrow (\bar{\nu}\nu, \mu^+\mu^-)$ are included, $BR(\tau \rightarrow 3\mu + \cancel{e}, 5\mu)$ of 10^{-9} can fall within the sensitivity of Belle II experiment in the search for the rare tau decays.

Acknowledgments

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Appendix

The Higgs Yukawa couplings in Eq. (10) is written as:

$$\begin{aligned}
-\mathcal{L}_h &= \left(\bar{\ell}_L, \bar{\Psi}_{\tau'L} \right) V_L \left(\begin{array}{c|c} \mathbf{m}_{\ell 3 \times 3} & \delta \mathbf{m}_1 \\ \hline 0 & 0 \end{array} \right) V_R^\dagger \left(\begin{array}{c} \ell_R \\ \Psi_{\tau'R} \end{array} \right) \frac{h}{v} + H.c., \\
&\approx \left(\bar{\ell}_L, \bar{\Psi}_{\tau'L} \right) \left(\begin{array}{c|c} \mathbf{m}_\ell - \delta \mathbf{m}_1 \epsilon_R^\dagger & \delta \mathbf{m}_1 \\ \hline 0 & \epsilon_L^\dagger \delta \mathbf{m}_1 \end{array} \right) \left(\begin{array}{c} \ell_R \\ \Psi_{\tau'R} \end{array} \right) \frac{h}{v} + H.c., \tag{32}
\end{aligned}$$

where we have applied the flavor mixing matrices of Eq. (8) in the second line, and $\delta \mathbf{m}_1 \epsilon_R^\dagger$ and $\epsilon_L^\dagger \delta \mathbf{m}_1$ are given by:

$$\delta \mathbf{m}_1 \epsilon_R^\dagger = \frac{v v_S}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_\mu y'_\mu \left(-\frac{c_\alpha s_\alpha}{m_{L1}} + \frac{c_\alpha s_\alpha}{m_{L2}} \right) & y_\mu y_\tau \left(\frac{c_\alpha^2}{m_{L1}} + \frac{s_\alpha^2}{m_{L2}} \right) \\ 0 & y'_\mu y'_\tau \left(\frac{s_\alpha^2}{m_{L1}} + \frac{c_\alpha^2}{m_{L2}} \right) & y_\tau y'_\tau \left(-\frac{c_\alpha s_\alpha}{m_{L1}} + \frac{c_\alpha s_\alpha}{m_{L2}} \right) \end{pmatrix}, \tag{33}$$

$$\epsilon_L^\dagger \delta \mathbf{m}_1 = \frac{v^2}{2} \begin{pmatrix} \frac{y_\mu^2 c_\alpha^2 + y_\tau'^2 s_\alpha^2}{m_{L1}} & \frac{y_\mu^2 - y_\tau'^2}{m_{L1}} c_\alpha s_\alpha \\ \frac{y_\mu^2 - y_\tau'^2}{m_{L2}} c_\alpha s_\alpha & \frac{y_\mu^2 s_\alpha^2 + y_\tau'^2 c_\alpha^2}{m_{L2}} \end{pmatrix}. \tag{34}$$

Hence, the Higgs couplings to the charged leptons can be decomposed as:

$$\begin{aligned}
-\mathcal{L}_h &= \frac{m_\ell}{v} \bar{\ell}_L \ell_R h + \frac{v_S c_\alpha s_\alpha}{2} \frac{m_{L2} - m_{L1}}{m_{L1} m_{L2}} (y_\mu y'_\mu \bar{\mu}_L \mu_R + y_\tau y'_\tau \bar{\tau}_L \tau_R) h \\
&- \frac{v_S y_\mu y_\tau}{2} \left(\frac{c_\alpha^2}{m_{L1}} + \frac{s_\alpha^2}{m_{L2}} \right) \bar{\mu}_L \tau_R h - \frac{v_S y'_\mu y'_\tau}{2} \left(\frac{s_\alpha^2}{m_{L1}} + \frac{c_\alpha^2}{m_{L2}} \right) \bar{\tau}_L \mu_R h \\
&+ \frac{y_\mu}{\sqrt{2}} \bar{\mu}_L (c_\alpha \tau'_R + s_\alpha \tau''_R) h + \frac{y'_\tau}{\sqrt{2}} \bar{\tau}_L (-s_\alpha \tau'_R + c_\alpha \tau''_R) h + \bar{\Psi}_{\tau'L} \epsilon_L^\dagger \delta \mathbf{m}_1 \Psi_{\tau'R} \frac{h}{v} + H.c. \tag{35}
\end{aligned}$$

Similarly, the ϕ_S Yukawa couplings in Eq. (10) is expressed as:

$$\begin{aligned}
-\mathcal{L}_{\phi_S} &= \left(\bar{\ell}_L, \bar{\Psi}_{\tau'L} \right) V_L \left(\begin{array}{c|c} 0 & 0 \\ \hline \delta \mathbf{m}_2^T & 0 \end{array} \right) V_R^\dagger \left(\begin{array}{c} \ell_R \\ \Psi_{\tau'R} \end{array} \right) \frac{\phi_S}{v_S}, \\
&\approx \left(\bar{\ell}_L, \bar{\Psi}_{\tau'L} \right) V_L \left(\begin{array}{c|c} -\epsilon_L \delta \mathbf{m}_2^T & 0 \\ \hline \delta \mathbf{m}_2^T & \delta \mathbf{m}_2^T \epsilon_R \end{array} \right) V_R^\dagger \left(\begin{array}{c} \ell_R \\ \Psi_{\tau'R} \end{array} \right) \frac{\phi_S}{v_S}, \tag{36}
\end{aligned}$$

where $\epsilon_L \delta \mathbf{m}_2^T = \delta \mathbf{m}_1 \epsilon_R^\dagger$, and $\mathbf{m}_2^T \epsilon_R$ is written as:

$$\mathbf{m}_2^T \epsilon_R = \frac{v_S}{2} \begin{pmatrix} \frac{y_\mu'^2 s_\alpha^2 + y_\tau^2 c_\alpha^2}{m_{L1}} & \frac{-y_\mu'^2 + y_\tau^2}{m_{L2}} c_\alpha s_\alpha \\ \frac{-y_\mu'^2 + y_\tau^2}{m_{L1}} c_\alpha s_\alpha & \frac{y_\mu'^2 c_\alpha^2 + y_\tau^2 s_\alpha^2}{m_{L2}} \end{pmatrix}. \tag{37}$$

Then, the ϕ_S Yukawa couplings to the charged leptons can be written as:

$$\begin{aligned}
-\mathcal{L}_{\phi_S} = & \frac{vc_\alpha s_\alpha}{2} \frac{m_{L2} - m_{L1}}{m_{L1}m_{L2}} (y_\mu y'_\mu \bar{\mu}_L \mu_R + y_\tau y'_\tau \bar{\tau}_L \tau_R) \phi_S - \frac{vy_\mu y_\tau}{2} \left(\frac{c_\alpha^2}{m_{L1}} + \frac{s_\alpha^2}{m_{L2}} \right) \bar{\mu}_L \tau_R \phi_S \\
& - \frac{vy'_\mu y'_\tau}{2} \left(\frac{s_\alpha^2}{m_{L1}} + \frac{c_\alpha^2}{m_{L2}} \right) \bar{\tau}_L \mu_R \phi_S + \frac{y'_\mu}{\sqrt{2}} (-s_\alpha \bar{\tau}'_L + c_\alpha \bar{\tau}''_L) \mu_R \phi_S \\
& + \frac{y_\tau}{\sqrt{2}} (c_\alpha \bar{\tau}'_L + s_\alpha \bar{\tau}''_L) \tau_R \phi_S + \bar{\Psi}_{\tau' L} \delta m_2^T \epsilon_R \Psi_{\tau' R} \frac{\phi_S}{v_S} + H.c.
\end{aligned} \tag{38}$$

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